CHAPTER 4

Prob. 4.1 $T_{sun} = 5800\,K$

Total brightness, $B_t = \frac{\sigma T^4}{\pi}$ from Eq. (4.27)
where $\sigma = 5.673 \times 10^{-8}\ W\,m^{-2}\,K^{-4}\,sr^{-1}$

$B_t = 2.044 \times 10^7\ W\,m^{-2}\,sr^{-1}$

Spectral brightness B_f is max. at f_m

$f_m = 5.87 \times 10^{10}\,T\ (Hz)$ Eq. (4.31)

$= 3.40 \times 10^{14}\ Hz$

$$B_1 = \int_{\frac{f_m}{2}}^{2f_m} B_f\, df = \frac{2}{c^2h^3}(kT)^4 \int_{\frac{\frac{f_m}{2}h}{kT}}^{\frac{2f_m h}{kT}} \frac{x^3}{e^x - 1}\, dx$$

where $x = \frac{hf}{kT}$

$2 f_m h / kT = 5.633 = x_2$

$\frac{1}{2} f_m h / kT = 1.408 = x_1$

$$B_1 = B_t - \frac{2}{c^2h^3}(kT)^4 \left[\int_0^{x_1} \frac{x^3}{e^x - 1}\, dx + \int_{x_2}^{\infty} \frac{x^3}{e^x - 1}\, dx \right]$$

$$\int_0^{x_1} \frac{x^3}{e^x - 1}\, dx = \sum_{n=1}^{\infty} \int_0^{x_1} x^3 e^{-nx}\, dx$$

$$= \sum_{n=1}^{\infty} \left[e^{-nx} \sum_{r=0}^{3} \frac{-3!}{n^{r+1}} \cdot \frac{x^{3-r}}{(3-r)!} \right] \Bigg|_0^{x_1}$$

$$= \sum_{n=1}^{\infty} \left[\frac{3!}{n^4} - e^{-nx_1} 3! \left(\frac{x_1^3}{3!\, n} + \frac{x_1^2}{2!\, n^2} + \frac{x_1}{1!\, n^3} + \frac{1}{n^4} \right) \right]$$

Prob. 4.1 (cont.) $\quad X_1 = 1.408$

$$\int_0^{x_1} \frac{x^3}{e^x-1}\,dx = 6\sum_{n=1}^{\infty}\left[n^{-4} - e^{-nx_1}\left(\frac{x_1^3}{6n} + \frac{x_1^2}{2n^2} + \frac{x_1}{n^3} + \frac{1}{n^4}\right)\right]$$

$$\sum_{n=1}^{\infty} n^{-4} = \frac{\pi^4}{90} = 1.0823$$

$$\sum_{n=1}^{\infty} e^{-nx_1}\left(\frac{x_1^3}{6n} + \frac{x_1^2}{2n^2} + \frac{x_1}{n^3} + \frac{1}{n^4}\right) \cong 1.03315$$

$$\therefore \int_0^{x_1} \frac{x^3}{e^x-1}\,dx \cong 0.295$$

$$\int_{x_2}^{\infty} \frac{x^3}{e^x-1}\,dx \cong \int_{x_2}^{\infty} x^3 e^{-x}\,dx \quad \text{from Weins Radiation Law}$$

$$\int_{x_2}^{\infty} x^3 e^{-x}\,dx = -e^{-x}\sum_{r=0}^{3}\frac{3!\,x^{3-r}}{(3-r)!}\Bigg|_{x_2}^{\infty}$$

$$= 3!\,e^{-x_2}\left[\frac{x_2^3}{3!} + \frac{x_2^2}{2!} + \frac{x_2}{1!} + 1\right] = e^{-x_2}\left[x_2^3 + 3x_2^2 + 6x_2 + 6\right]$$

$$x_2 = 5.633 \rightarrow \int_{x_2}^{\infty} \frac{x^3}{e^x-1}\,dx \cong 1.122$$

$$B_t = \frac{2}{c^2h^3}(kT)^4\,\frac{6\pi^4}{90}$$

$$B_1 = \frac{2}{c^2h^3}(kT^4)\left[\frac{6\pi^4}{90} - 0.295 - 1.122\right]$$

$$= \frac{2}{c^2h^3}(kT^4)(5.077)$$

$$B_1/B_t = \frac{5.077}{6\pi^4/90} = 0.782$$

$\therefore$ 78.2% of B_t is between $\frac{f_m}{2}$ and $2f_m$.

Prob. 4.3 $\quad \Theta_s = 0.5°$, $\lambda = 1$ cm , $T_A' = 1174$ K

$T_o = 300$ K , $\eta_\ell = 0.8$, $A_r = 0.4$ m^2

$$T_A' = \eta_\ell T_A + (1-\eta_\ell) T_o$$

a) $$T_A = \frac{T_A' - (1-\eta_\ell)T_o}{\eta_\ell} = 1392.5 \text{ K}$$

$$T_A = \frac{\Omega_s}{\Omega_p} T_s \quad \text{for } \Omega_p \gg \Omega_s$$

$$\Omega_s = \pi\left(\frac{\Theta_s}{2}\right)^2 = 5.98 \times 10^{-5} \text{ sr}$$

$$\Omega_p = \frac{\lambda^2}{A_r} = 2.5 \times 10^{-4} \text{ sr}$$

$$\Omega_p \gg \Omega_s$$

$$T_s = \frac{\Omega_p}{\Omega_s} T_A = 5820 \text{ K}$$

b) $\eta_\ell = 1$ Assumed

$$T_A' = T_A = 1174$$

$$\rightarrow T_s = \frac{\Omega_p}{\Omega_s} T_A' = 4908 \text{ K}$$

$$\text{percent error} = \frac{5820 - 4908}{5820} \times 100 = 15.7\%$$

c) $A_r = 3$ m^2 $\rightarrow \Omega_p = 3.33 \times 10^{-5}$ sr

$\Omega_p < \Omega_s \rightarrow T_A = T_s$

$\eta_\ell = 1 \rightarrow T_A' = T_s = 5820$ K

Prob. 4.4 $100\,K < T_{ML} < 300\,K$ $\eta_l = 0.9$

$100\,K < T_{SL} < 200\,K$ T_0 known

$$T_A' = \eta_l \eta_M T_{ML} + \eta_l (1-\eta_M) T_{SL} + (1-\eta_l) T_0$$

from Eq. (4.62)

$$T_A' - (1-\eta_l) T_0 = \eta_l \left[\eta_M T_{ML} + (1-\eta_M) T_{SL} \right]$$

would like the estimated terrain temp, T_{ML}^* to be within 0.97 to 1.03 of the true terrain temp., T_{ML}, or $\frac{|T_{ML} - T_{ML}^*|}{T_{ML}} \le 0.03$

$$T_{ML} = \left[T_A' - \eta_l (1-\eta_M) T_{SL} - (1-\eta_l) T_0 \right] / (\eta_l \eta_M)$$

ASSUME A VALUE FOR T_{SL} OF T_{SL}^*

$$T_{ML}^* = \left[\eta_l \left(\eta_M T_{ML} + (1-\eta_M) T_{SL} \right) - \eta_l (1-\eta_M) T_{SL}^* \right] / (\eta_l \eta_M)$$

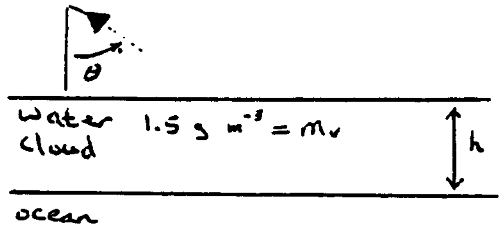
$$T_{ML}^* = T_{ML} + (T_{SL} - T_{SL}^*) \frac{1-\eta_M}{\eta_M}$$

$$\frac{|T_{ML} - T_{ML}^*|}{T_{ML}} = \frac{\left| (T_{SL} - T_{SL}^*) \frac{1-\eta_M}{\eta_M} \right|}{T_{ML}} \le 0.03$$

WORST CASE: T_{ML} = minimum = 100 K, and $T_{SL} - T_{SL}^*$ = maximum = 100 K

$$\therefore \quad \frac{1-\eta_M}{\eta_M} \le 0.03 \rightarrow \eta_M \ge 0.971$$

Prob. 4.5



$h = 2\,km$, $\theta = 0$

$\kappa_a \cong 2.4 \times 10^{-4} f^{1.95} m_v$, $Np\ km^{-1}$

for ocean , $T_{AP}(0;0) = 150\,K$

for cloud, $T_0 = 275\,K$

$1\,GHz \le f \le 30\,GHz$

$$T_{AP}(\theta;H) = T_{AP}(\theta;0)\, e^{-\tau(0,H)\sec\theta} + T_{up}(\theta;H)$$

from Eq. (4.100)

where $\tau(z',H) = \int_{z'}^{H} \kappa_a(z)\,dz$

and $T_{up}(\theta;H) = \sec\theta \int_0^H \kappa_a(z')\,T(z')\, e^{-\tau(z',H)\sec\theta}\, dz'$

since κ_a is indep. of z, $\tau(z',H) = \kappa_a(H - z')$

also $T(z') = \text{constant} = 275\,K$

$$T_{up}(\theta;H) = T_0 \sec\theta\, \kappa_a \int_0^H \exp[-\sec\theta\, \kappa_a (H - z')]\, dz'$$

$$= \kappa_a T_0 \sec\theta \exp[-H \cdot \kappa_a \cdot \sec\theta] \left. \frac{\exp[\kappa_a \cdot \sec\theta \cdot z']}{\kappa_a \cdot \sec\theta} \right|_0^H$$

$$= T_0 \left(1 - e^{-H \cdot \kappa_a \cdot \sec\theta}\right)$$

$$T_{AP}(\theta;H) = T_{AP}(\theta;0)\, e^{-H \cdot \kappa_a \cdot \sec\theta} + T_0\left(1 - e^{-H \cdot \kappa_a \cdot \sec\theta}\right)$$

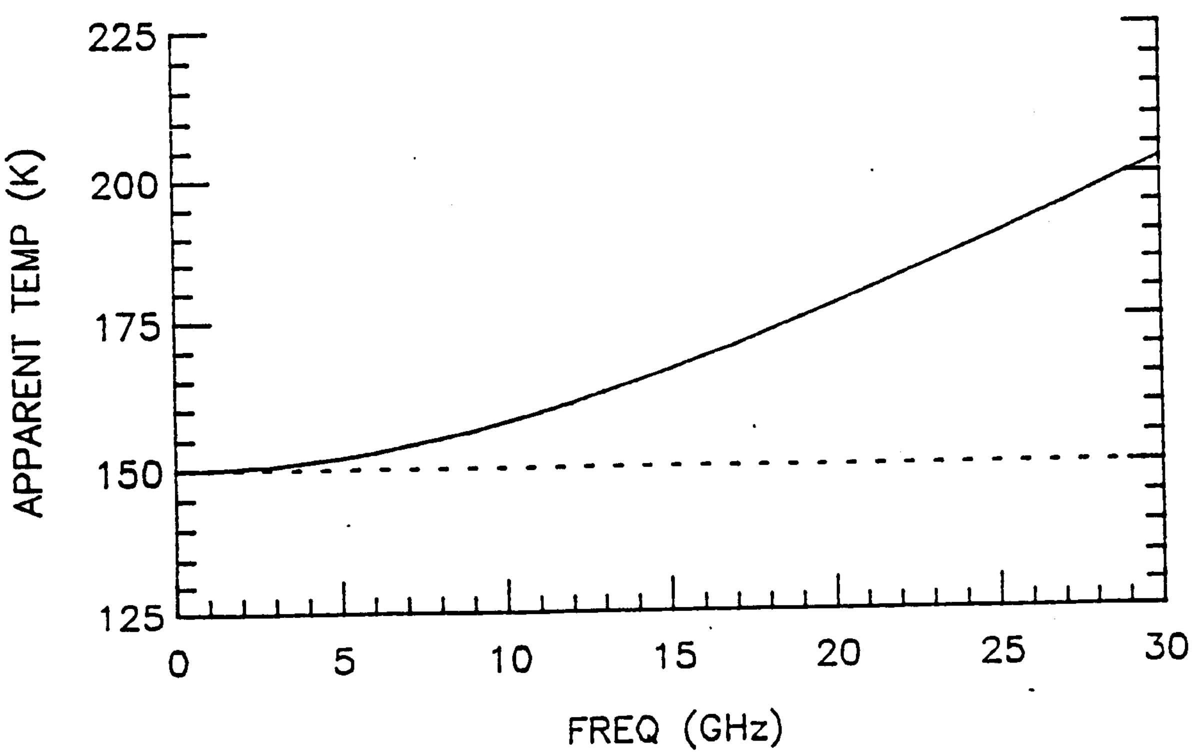
$$T_{AP}(\theta;H) = T_{AP}(\theta;0) \frac{1}{L_a(\theta;H)} + T_0 \left(1 - \frac{1}{L_a(\theta;H)}\right)$$

where $L_a(\theta;H) = e^{\tau(0,H)\sec\theta} = e^{H \cdot \kappa_a \cdot \sec\theta}$

Prob. 4.5 (cont.) $H = 2$ km, $\theta = 0^\circ$, $\kappa_a = 2.4 \times 10^{-4} f^{1.95} m_v$

$m_v = 1.5\ g/m^3$, $T_{AP}(0;0) = 150$ K, $T_0 = 275$ K

f (GHz)	κ_a (N_p km^{-1})	L_a	$T_{AP}(0;H)$ (K)
1	3.6×10^{-4}	1.00072	150.09
5	8.3×10^{-3}	1.017	152.06
10	3.2×10^{-2}	1.066	157.77
15	7.1×10^{-2}	1.15	166.49
20	1.2×10^{-1}	1.28	177.45
25	1.9×10^{-1}	1.47	189.78
30	2.7×10^{-1}	1.73	202.64

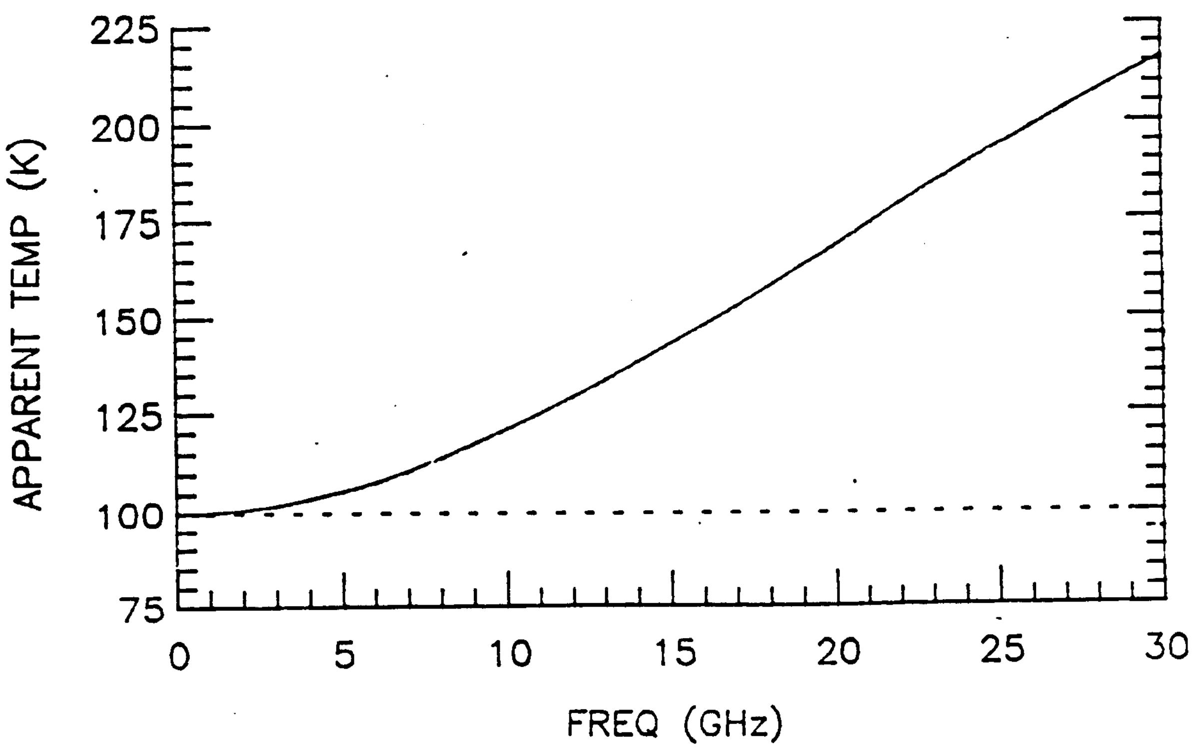


Prob. 4.6 $\theta = 60°$, $T_{AP}(60°;0) = 100\,K$

See Prob. 4.5

replace θ with $60°$, $T_{AP}(\theta;0)$ with $100\,K$

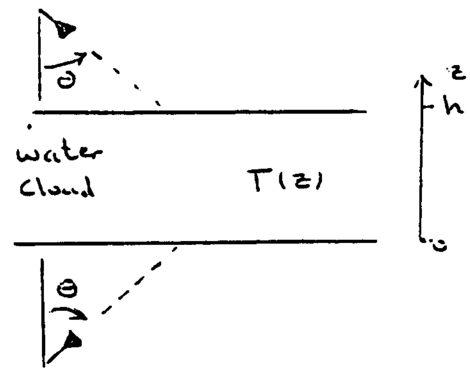
f (GHz)	L_a	$T_{AP}(60°;H)$ (K)
1	1.0014	100.25
5	1.0338	105.72
10	1.1369	121.08
15	1.3271	143.13
20	1.6419	168.42
25	2.1516	193.66
30	2.9842	216.35



Prob. 4.7

$h = 2$ km

$\theta = 60°$



$$T(z) = 300\,(1 - 4\times10^{-2} z) \rightarrow T(2\,km) = 276\,K$$
$$T(0\,km) = 300\,K$$

$$T_{up}(\theta; H) = \sec\theta \int_0^H k_a(z')\,T(z')\,e^{-\tau(z',H)\sec\theta}\,dz'$$

$$T_{DN}(\theta; H) = \sec\theta \int_0^H k_a(z')\,T(z')\,e^{-\tau(0,z')\sec\theta}\,dz'$$

where

$$\tau(a,b) = \int_a^b k_a(z)\,dz$$

for k_a = constant

$$\tau(a,b) = k_a(b-a)$$

$$T_{up} = \sec\theta\, k_a \int_0^H T(z')\,e^{-k_a(H-z')\sec\theta}\,dz'$$

$$= \sec\theta\, k_a\, e^{-k_a\cdot H\cdot\sec\theta} \int_0^H T(z')\,e^{k_a z'\sec\theta}\,dz'$$

$$T(z') = T_0\,(1 - \alpha z')$$

$$T_{up} = \sec\theta\, k_a\, e^{-k_a\cdot H\cdot\sec\theta}\cdot T_0\left[\int_0^H e^{k_a\cdot z'\cdot\sec\theta}\,dz' - \alpha\int_0^H z' e^{k_a\cdot z'\cdot\sec\theta}\,dz'\right]$$

$$= \sec\theta\cdot k_a\cdot T_0\cdot e^{-k_a\cdot H\cdot\sec\theta}\left[\frac{e^{k_a\cdot z'\cdot\sec\theta}}{k_a\cdot\sec\theta}\Bigg|_0^H - \frac{\alpha\, e^{k_a\cdot z'\cdot\sec\theta}}{(k_a\cdot\sec\theta)^2}\cdot (k_a\cdot\sec\theta\cdot z' - 1)\Bigg|_0^H\right]$$

Prob. 4.7 (cont.)

$$T_{up}(\theta;H) = T_o\, e^{-k_a \cdot H \cdot \sec\theta}\Big[\left(e^{k_a \cdot H \cdot \sec\theta} - 1\right) - \Big\{\alpha H e^{k_a \cdot H \cdot \sec\theta} - \alpha\left(e^{k_a \cdot \sec\theta \cdot H} - 1\right)/k_a \sec\theta\Big\}\Big]$$

$$= T_o\left[\left(1 - e^{-k_a \cdot H \cdot \sec\theta}\right) - \alpha H + \frac{\alpha}{k_a \sec\theta}\left(1 - e^{-k_a \cdot H \cdot \sec\theta}\right)\right]$$

$$T_{up}(\theta;H) = T_o\left[\left(1 - e^{-k_a \cdot H \cdot \sec\theta}\right)\left(1 + \frac{\alpha}{k_a \sec\theta}\right) - \alpha H\right]$$

$$T_{DN}(\theta;H) = k_a \cdot T_o \cdot \sec\theta \int_0^H (1 - \alpha z')\, e^{-k_a \cdot z' \cdot \sec\theta}\, dz'$$

$$= k_a \cdot T_o \cdot \sec\theta\left[\int_0^H e^{-k_a \cdot z' \cdot \sec\theta}\, dz' - \alpha \int_0^H z' e^{-k_a \cdot z' \cdot \sec\theta}\, dz'\right]$$

$$= T_o \cdot k_a \cdot \sec\theta\left[\frac{e^{-k_a \cdot z' \cdot \sec\theta}}{-k_a \cdot \sec\theta}\Bigg|_0^H - \alpha\, \frac{e^{-k_a \cdot z' \cdot \sec\theta}}{(-k_a \cdot \sec\theta)^2}\left(-k_a \cdot \sec\theta \cdot z' - 1\right)\Bigg|_0^H\right]$$

$$= T_o\left(1 - e^{-k_a \cdot H \cdot \sec\theta} + \alpha\, \frac{e^{-k_a \cdot H \cdot \sec\theta}}{k_a \cdot \sec\theta}\left(k_a \cdot \sec\theta \cdot H + 1\right) - \alpha\, \frac{1}{k_a \sec\theta}\right)$$

$$T_{DN}(\theta, H) = T_o\left[\left(1 - e^{-k_a \cdot H \cdot \sec\theta}\right)\left(1 - \frac{\alpha}{k_a \sec\theta}\right) + \alpha \cdot H \cdot e^{-k_a \cdot H \cdot \sec\theta}\right]$$

Now $\theta = 60^\circ$, $H = 2\,\text{km}$, $T_o = 300\,\text{K}$, $\alpha = 4\times10^{-2}\ \text{K km}^{-1}$
and $k_a =$ (a) 10^{-4}, (b) 10^{-3}, (c) 10^{-2} Np km^{-1}

a) $T_{up} = 0.1152\,\text{K}$, $T_{DN} = 0.1152\,\text{K}$

b) $T_{up} = 1.1497\,\text{K}$, $T_{DN} = 1.1497\,\text{K}$

Prob. 4.7 (cont.)

c) $T_{up} = 11.290\ K$, $T_{DN} = 11.296\ K$

In all but the last case, $T_{up} = T_{DN}$.
If however $K_a \to \infty$ then clearly $T_{up} \neq T_{DN}$
since $T_{up} = T(\text{top}) = 276\ K$ and $T_{DN} = T(\text{bottom}) = 300\ K$

Prob. 4.8 $T_{AP}(\theta; H) = T_{up}(\theta; H) = \sec\theta \int_0^H K_a(z')\, T(z')\, e^{-\tau(z',H)\sec\theta}\, dz'$

$$e^{-\tau(z',H)} = \int_{z'}^{H} K_a(z)\,dz$$

We want $T_{AP} \geq 0.99\, T_o$, for $T(z') = T_o$
$K_a(z') = K_{ao}$
$\theta = 0°$

$$T_{AP}(0; H) = T_o\left[1 - e^{-\tau(0,H)}\right] \quad \text{from Eq. (4.98)}$$

$$1 - e^{-\tau(0,H)} \geq 0.99$$

$$e^{-\tau(0,H)} \leq 0.01$$

$$\tau(0,H) \geq 4.61$$

$$\tau(0,H) = H \cdot K_{ao}$$

$$K_{ao} \geq \frac{4.61}{H}$$

Prob. <u>4.10</u> SKYLAB RADIOMETER

$\theta = 0°$, $\lambda = 21$ cm

$$F_n(\theta_a) = \exp[-2.77(\theta_a/\beta)^2]$$

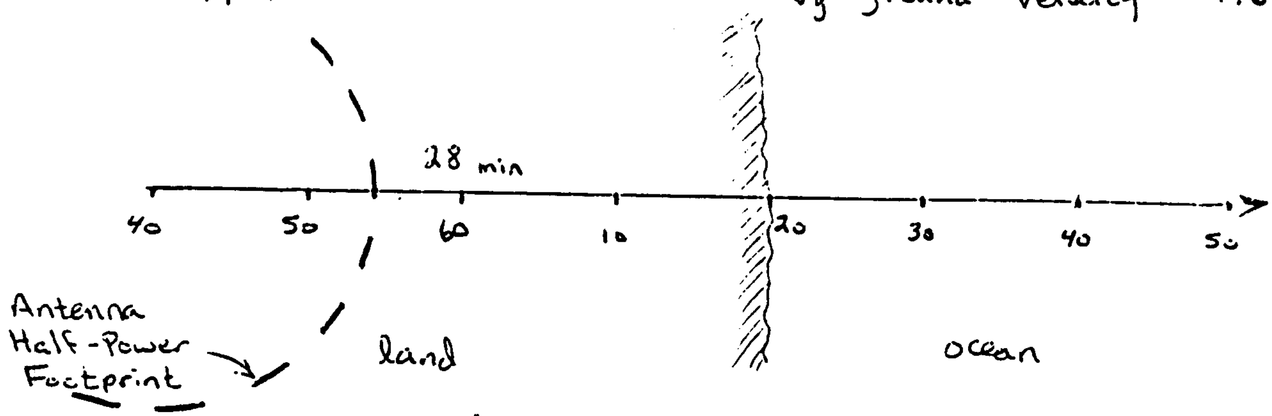
$\beta = 15°$

T_{AP} (land) = 250 K

T_{AP} (ocean) = 90 K

h = sensor altitude = 435 km

V_g = ground velocity = 7.65 km s^{-1}



Assumptions -

Antenna is lossless

Atmosphere is lossless

Sidelobe contributions are negligible

T_{AP} is incidence angle independent

Effective footprint has $\beta' = 30°$

Footprint diameter $= 2 \cdot h \cdot \tan \beta'/2 = 233$ km

time required to move $\perp$ footprint $= \dfrac{d}{V_g} = \dfrac{233}{7.65} = 30.47$ sec

Prob. 4.10 (cont.)

footprint just 'touches' coast at

28 min 20 sec $- \frac{30.47}{2}$ sec or 28 min 4.76 sec

on the ocean side, the foot print just 'touches' the coast at 28 min 20 sec $+ \frac{30.47}{2}$ sec or 28 min 35.24 sec

Therefore for $t < 28$ min 4.76 sec, $T_{AP} = 250\,K$

and for $t > 28$ min 35.24 sec, $T_{AP} = 90\,K$

for 28 min 4.76 sec $\leq t \leq 28$ min 35.24 sec

$$90K \leq T_{AP} \leq 250K$$

Since there is only one main lobe

$$T_A = T_{ml} = \frac{1}{\Omega_M} \iint_{\substack{\text{main}\\ \text{lobe}}} T_{AP}(\theta_a, \phi) F_n(\theta_a, \phi)\, d\Omega_a$$

where $\Omega_M = \iint_{\substack{\text{main}\\ \text{lobe}}} F_n(\theta_a, \phi)\, d\Omega_a$

$$\Omega_m = \int_0^{2\pi}\int_0^{\pi} F_n(\theta_a, \phi) \sin\theta_a \, d\theta_a \, d\phi$$

limit θ_a to a maximum of 15° $(\pi/12)$

$\rightarrow \sin\theta_a \approx \theta_a$

Prob <u>4.10 (cont.)</u> $\beta = 15° = \frac{\pi}{12}$ radians

$$\Omega_m = \int_0^{2\pi}\int_0^{\frac{\pi}{12}} \theta_a \exp\left[-2.77(\theta_a/\beta)^2\right] d\theta_a d\phi$$

$$= 2\pi \left[\exp\{-2.77(\theta_a/\beta)^2\} / (2\cdot -2.77/\beta^2) \right]_0^{\pi/12}$$

$$\Omega_m = \frac{\pi\beta^2}{2.77}\left[1 - 0.06266\right] = 0.07286$$

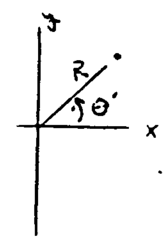
$$\Omega_m^{-1} = 13.725$$

$$J = \int_0^{2\pi}\int_0^{\frac{\pi}{12}} T_{ap}(\theta_a, \phi)\, F_n(\theta_a) \sin\theta_a d\theta_a d\phi$$

Change coordinates to rectangular (x, y)

$x = R\cos\theta'$ $\quad R = \sqrt{x^2+y^2}$

$y = R\sin\theta'$ $\quad \theta' = \tan^{-1}(y/x)$

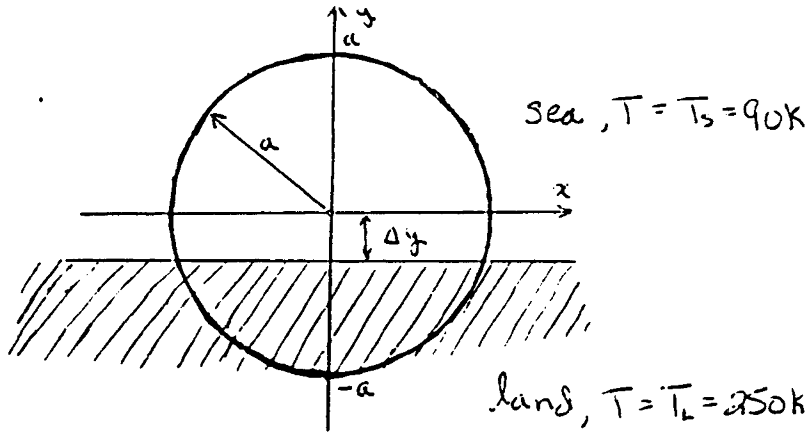


$$d\Omega_a = \frac{\cos\theta_1}{r^2}\, dx\, dy$$

where θ_1 is the local incidence angle and r is the distance to the antenna (see Fig. 4.17 from text)

For our case, i.e., nadir viewing, small β, $\theta_1 = \theta_a$, for small θ_a, $\cos\theta_a \approx 1$, $r \approx h$, also $\sin\theta_a \approx \theta_a$ and $\theta_a \approx \frac{R}{h} = \frac{\sqrt{x^2+y^2}}{h}$

Prob. 4.10 (cont.)



$x^2 + y^2 = a^2$

$x = \pm\sqrt{a^2 - y^2}$

when $\Delta y = 0$, $T_A = \frac{1}{2}T_L + \frac{1}{2}T_S = \frac{1}{2}(250+90) = 170\,K$

$\Delta y > 0$, $T_A = b\,T_L + (1-b)T_S < 170\,K$ but $T_A \geq 90\,K$

where $0 \leq b < \frac{1}{2}$

$\Delta y < 0$, $T_A = b\,T_L + (1-b)T_S > 170\,K$ but $T_A \leq 250\,K$

where $\frac{1}{2} < b \leq 1$

due to the symmetry, we need only compute one case, so we choose $\Delta y > 0$, $0 \leq b < \frac{1}{2}$

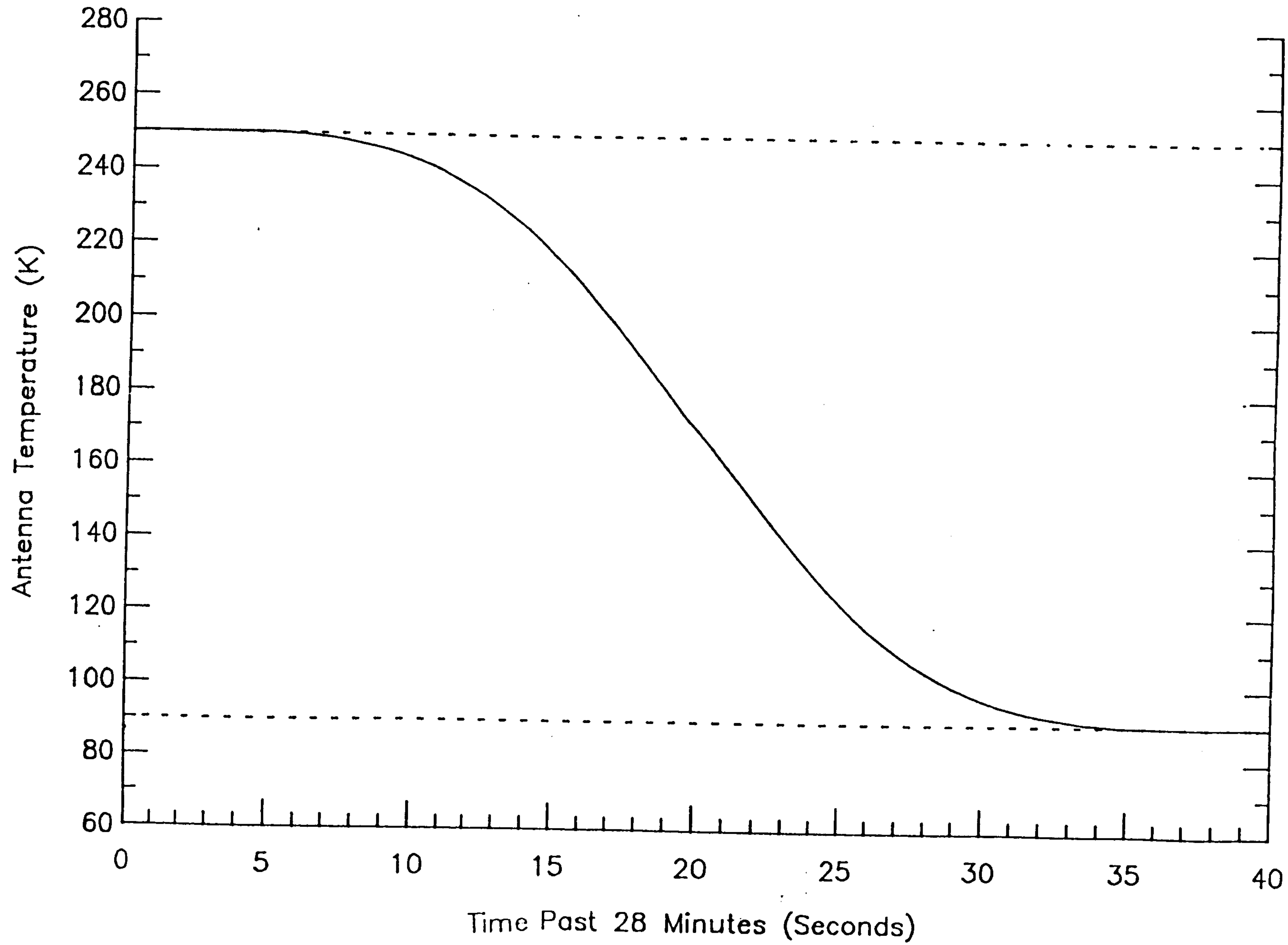
$$b = \frac{1}{\Omega_m}\int_{-a}^{-\Delta y} 2\cdot\int_0^{\sqrt{a^2-y^2}} \frac{1}{h^2}\, \exp\left[-2.77\cdot\frac{x^2+y^2}{h^2\beta^2}\right] dx\,dy$$

$$= \frac{2}{h^2\Omega_m}\int_{-a}^{-\Delta y} \exp\left[-2.77\left(\frac{y}{h\beta}\right)^2\right]\int_0^{\sqrt{a^2-y^2}} \exp\left[-2.77\left(\frac{x}{h\beta}\right)^2\right] dx\,dy$$

$\Delta y = V_g(t - t_o)$, $t_o = 28$ min 20 sec

$a = h\cdot\tan\left(\frac{\beta'}{2}\right) = 116.6$ km

Numerical integration follows using method described in Prob. 4.9. See the program which follows.



280
260
240
220
200
180
160
140
120
100
80
60
Antenna Temperature (K)
0
5
10
15
20
25
30
35
40
Time Past 28 Minutes (Seconds)