CHAPTER 3

Prob. 3.1 FAR-ZONE ELECTRIC FIELD

$$E_\theta = \begin{cases} \dfrac{e^{-jkr}}{r}\cos^2\theta, & 0 \le \theta \le \pi/2 \\ 0, & \text{elsewhere} \end{cases}$$

$$E_\phi = 0$$

$$E_\theta = \frac{e^{-jkr}}{r} f_1(\theta,\phi) \longrightarrow f_1(\theta,\phi) = \cos^2\theta$$

$$E_\phi = \frac{e^{-jkr}}{r} f_2(\theta,\phi) \longrightarrow f_2(\theta,\phi) = 0$$

$$S_r = \frac{1}{2\eta r^2}\left(|f_1(\theta,\phi)|^2 + |f_2(\theta,\phi)|^2\right)$$

$$= \frac{1}{2\eta r^2}\cos^4\theta$$

$$F(\theta,\phi) = r^2 S_r = \frac{1}{2\eta}\cos^4\theta$$

$$F_n(\theta,\phi) = \frac{F(\theta,\phi)}{F(\theta,\phi)_{max}} = \cos^4\theta$$

a) $$\Omega_p = \iint_{4\pi} F_n(\theta,\phi)\,d\Omega = \int_0^{2\pi} d\phi \int_0^{\pi/2} \cos^4\theta \sin\theta\, d\theta$$

$$= 2\pi\left[-\frac{\cos^5\theta}{5}\right]_0^{\pi/2} = \frac{2}{5}\pi$$

b) $$\Omega_M = \iint_{\substack{\text{main}\\ \text{lobe}}} F_n(\theta,\phi)\,d\Omega = \int_0^{2\pi} d\phi \int_0^{\theta_{null}} F_n(\theta,\phi)\sin\theta\, d\theta$$

$$= 2\pi \int_0^{\pi/2} \cos^4\theta \sin\theta\, d\theta = \Omega_p = \frac{2}{5}\pi$$

Prob. 3.1 (cont.)

c) Symmetry in $\phi \rightarrow \beta_{xz} = \beta_{yz} = 2\theta_{1/2}$

$F_n(\theta_{1/2}, \phi) = 1/2 \rightarrow \cos^4 \theta_{1/2} = 1/2$, $\theta_{1/2} = 32.77°$

$\beta_{xz} = \beta_{yz} = 65.53° = 1.14$ rad

d) $D_o = \frac{4\pi}{\Omega_p} = \frac{4\pi}{\frac{2}{5}\pi} = 10$

e) $\Omega_p \cong \beta_{xz}\beta_{yz} = 1.3$

$D_o = \frac{4\pi}{\Omega_p} = 9.66$ (~ 10, good approximation)

f) $\lambda = 30$ cm

$A_{eff} = D_o \frac{\lambda^2}{4\pi} = \frac{10\,(0.3)^2}{4\pi} = 7.1 \times 10^{-2}\ m^2$

Prob. 3.2 $P_{in} = 20W$, $\eta_l = 0.85$

$P_o = P_{in} \cdot \eta_l = 17\,W$

$P_r = \frac{P_t}{4\pi R^2} G_t A_r$, $\frac{A_r}{R^2} = \Omega_r$

$P_{r\,max} = \frac{P_t G_o}{4\pi} \Omega_r = \frac{P_o D_o}{4\pi} \Omega_r$

$P_{r\,max}/\Omega_r = 1000\ W Sr^{-1} = \frac{17 \cdot D_o}{4\pi}$, $\rightarrow D_o = 739 = 28.7\,dB$

Prob. 3.3 $f = 6\,GHz \rightarrow \lambda = 5\,cm$

$d_E = 10\,m$, $d_s = 1\,m$, $R = 35{,}860\,km$

$P_t = 1\,kW$, $\eta_a = 1$, $\eta_\ell = 1$

from (3.36) $P_r/P_t = \eta_r \eta_t \, A_t A_r / (\lambda^2 R^2)$

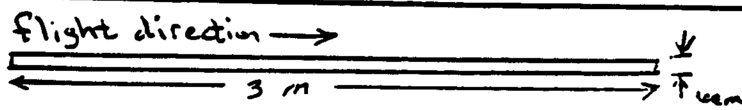
here $\eta_r = \eta_t = 1$

$A_t = \pi (d_s/2)^2 = 0.785\ m^2$

$A_r = \pi (d_E/2)^2 = 78.5\ m^2$

$P_r = (10^3)\ (0.785)(78.5) / [(0.05)^2 (35.86 \times 10^6)^2]$

$P_r = 1.92 \times 10^{-8}\ W$



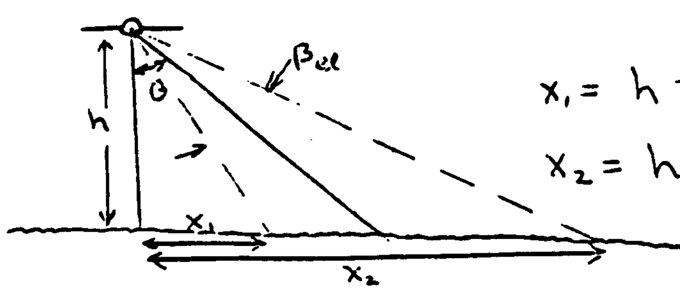
Prob. 3.4

$\lambda = 3.2\,cm$

a) $\beta_{az} \cong \dfrac{\lambda}{\ell_{az}} = \dfrac{3.2}{300} = 0.0107\ rad = 0.611°$

$\beta_{el} \cong \dfrac{\lambda}{\ell_{el}} = \dfrac{3.2}{6} = 0.533\ rad = 30.6°$

b)



$x_1 = h \tan(\theta - \tfrac{1}{2}\beta_{el})$

$x_2 = h \tan(\theta + \tfrac{1}{2}\beta_{el})$

Prob. 3.4 (cont.)

$$\theta = 50^\circ \; , \quad h = 3\,\text{km} \; , \quad \beta_{el} = 30.6^\circ$$

$$x_1 = 3 \tan(50 - 15.3) = 2.08 \text{ km}$$

$$x_2 = 3 \tan(50 + 15.3) = 6.52 \text{ km}$$

$$\Delta x = x_2 - x_1 = 4.44 \text{ km} = \text{ground swathwidth}$$

Prob. 3.5 Rectangular Aperture

a) $$E_a(x_a, y_a) = \begin{cases} E_0 \cos\left(\frac{\pi x_a}{a}\right) & , \text{ for } |x_a| \leq a/2 \text{ and } |y_a| \leq b/2 \\ 0 & , \text{ otherwise} \end{cases}$$

far field $E(R, \Theta, \phi) = \dfrac{j e^{-jkR}}{\lambda R} h(\theta, \phi)$ from (3.78)

where

$$h(\theta, \phi) = \iint_{-\infty}^{\infty} E_a(x_a, y_a) \exp[jk\sin\theta\,(x_a \cos\phi + y_a \sin\phi)]\, dx_a\, dy_a$$

from (3.79)

$$h(\theta, \phi) = \int_{-b/2}^{b/2} \int_{-a/2}^{a/2} E_0 \cos\left(\frac{\pi x_a}{a}\right) \exp[jk\sin\theta\,(x_a \cos\phi + y_a \sin\phi)]\, dx_a\, dy_a$$

$$= E_0 \int_{-a/2}^{a/2} \cos\left(\frac{\pi x_a}{a}\right) e^{jk\sin\theta\, x_a \cos\phi}\, dx_a \cdot \int_{-b/2}^{b/2} e^{jk\sin\theta\, y_a \sin\phi}\, dy_a$$

$$= E_0 \int_{-a/2}^{a/2} \frac{1}{2} \left[e^{j\left[\frac{\pi}{a} + k\sin\theta\cos\phi\right] x_a} + e^{j\left[-\frac{\pi}{a} + k\sin\theta\cos\phi\right] x_a} \right] dx_a \cdot$$

$$\int_{-b/2}^{b/2} e^{jk\sin\theta\sin\phi\, y_a}\, dy_a$$

Prob. 3.6 Gaussian illumination

$$E_a(x_a, y_a) = \begin{cases} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-x_1^2}{2\sigma^2}\right), & \text{for } |x_1| \le 1 \text{ and } |y_1| \le 1 \\ 0, & \text{otherwise} \end{cases}$$

where $x_1 = \frac{2x_a}{a}$, $y_1 = \frac{2y_a}{b}$

FAR-FIELD RADIATION PATTERN

$$E(R, \theta, \phi) = j\frac{e^{-jkR}}{\lambda R} h(\theta, \phi)$$

where

$$h(\theta,\phi) = \iint_{-\infty}^{\infty} E_a(x_a, y_a) \exp[jk\sin\theta(x_a\cos\phi + y_a\sin\phi)]\,dx_a\,dy_a$$

$$h(\theta,\phi) = \int_{-\frac{b}{2}}^{\frac{b}{2}}\int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{2x_a^2}{a^2\sigma^2}\right) \exp\{jk\sin\theta(x_a\cos\phi + y_a\sin\phi)\}\,dx_a\,dy_a$$

$$= \int_{-\frac{b}{2}}^{\frac{b}{2}} \exp[jk\sin\theta\sin\phi\, y_a]\,dy_a \cdot \int_{-a/2}^{a/2} \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{-2x_a^2}{a^2\sigma^2} + jk\sin\theta\cos\phi\, x_a\right) dx_a$$

BUT

$$\frac{-2x_a^2}{a^2\sigma^2} + jk\sin\theta\cos\phi\, x_a = -\left\{\left(\frac{\sqrt{2}\,x_a}{a\sigma} - j\frac{a\sigma k\sin\theta\cos\phi}{2\sqrt{2}}\right)^2 + \frac{a^2\sigma^2k^2\sin^2\theta\cos^2\phi}{8}\right\}$$

$$h(\theta,\phi) = b\,\text{sinc}\left(\frac{b}{\lambda}\sin\theta\sin\phi\right) e^{-\frac{a^2\sigma^2k^2\sin^2\theta\cos^2\phi}{8}} \cdot \int_{-a/2}^{a/2} \frac{1}{\sigma\sqrt{2\pi}} \exp\left[-\left(\frac{\sqrt{2}\,x_a}{a\sigma} - j\frac{k\sin\theta\cos\phi\,\sigma a}{2\sqrt{2}}\right)^2\right] dx_a$$

Prob. <u>3.6 (cont.)</u> let $u = \frac{\sqrt{2}\, x_a}{a\sigma} - j\frac{k\sin\theta\cos\phi\,\sigma a}{2\sqrt{2}}$

$$du = \frac{\sqrt{2}}{a\sigma}$$

$$h(\theta,\phi) = b\,\mathrm{sinc}\left(\frac{b}{\lambda}\sin\theta\sin\phi\right) e^{-\frac{a^2\sigma^2 k^2 \sin^2\theta\cos^2\phi}{8}} \cdot$$

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{\frac{-1}{\sigma\sqrt{2}} - j\frac{k\sin\theta\cos\phi\,\sigma a}{2\sqrt{2}}}^{\frac{1}{\sigma\sqrt{2}} - j\frac{k\sin\theta\cos\phi\,\sigma a}{2\sqrt{2}}} e^{-u^2} \frac{a\sigma}{\sqrt{2}}\, du$$

But $\mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-t^2}\,dt$, so

$$h(\theta,\phi) = \frac{ba}{4}\,\mathrm{sinc}\left(\frac{b}{\lambda}\sin\theta\sin\phi\right) e^{-\frac{k^2\sin^2\theta\cos^2\phi\,\sigma^2 a^2}{8}} \cdot$$

$$\left\{\mathrm{erf}\left(\frac{1}{\sigma\sqrt{2}} - j\frac{k\sin\theta\cos\phi\,\sigma a}{2\sqrt{2}}\right) - \mathrm{erf}\left(\frac{-1}{\sigma\sqrt{2}} - j\frac{k\sin\theta\cos\phi\,\sigma a}{2\sqrt{2}}\right)\right\}$$

$$F_n(\theta,\phi) = \frac{|h(\theta,\phi)|^2}{|h(0,\phi)|^2}$$

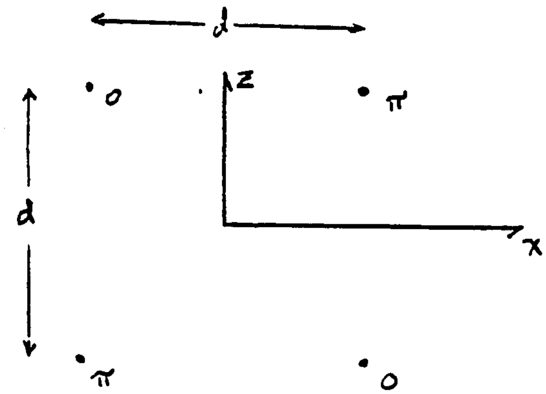
$$h(0,\phi) = \frac{ba}{4}\left\{\mathrm{erf}\left(\frac{1}{\sigma\sqrt{2}}\right) - \mathrm{erf}\left(\frac{-1}{\sigma\sqrt{2}}\right)\right\}$$

$$= \frac{ba}{2}\,\mathrm{erf}\left(\frac{1}{\sigma\sqrt{2}}\right)$$

$$F_n(\theta,\phi) = \frac{\mathrm{sinc}^2\left(\frac{b}{\lambda}\sin\theta\sin\phi\right) e^{-\frac{k^2\sin^2\theta\cos^2\phi\,\sigma^2 a^2}{4}}}{\left|4\,\mathrm{erf}\left(\frac{1}{\sigma\sqrt{2}}\right)\right|^2} \cdot$$

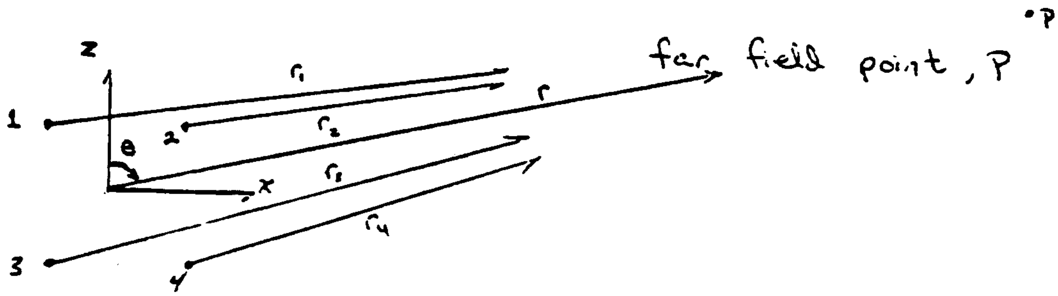
$$\left|\mathrm{erf}\left(\frac{1}{\sigma\sqrt{2}} - j\frac{k\sin\theta\cos\phi\,\sigma a}{2\sqrt{2}}\right) - \mathrm{erf}\left(\frac{-1}{\sigma\sqrt{2}} - j\frac{k\sin\theta\cos\phi\,\sigma a}{2\sqrt{2}}\right)\right|$$

Prob. 3.13



$d = \lambda/2$

equal amplitudes



$$F_a(\theta,\phi) = \left|\sum_{i=1}^{N} A_i e^{jkr_i}\right|^2 , \quad A_i = a_i e^{j\psi_i}$$

Here $a_i = 1$, $i = 1,2,3,4$

$\psi_2 = \psi_3 = \pi$, $\psi_1 = \psi_4 = 0$

$$r_1 = r + \frac{d}{2}\cos\phi\sin\theta - \frac{d}{2}\cos\theta$$
$$r_2 = r - \frac{d}{2}\cos\phi\sin\theta - \frac{d}{2}\cos\theta$$
$$r_3 = r + \frac{d}{2}\cos\phi\sin\theta + \frac{d}{2}\cos\theta$$
$$r_4 = r - \frac{d}{2}\cos\phi\sin\theta + \frac{d}{2}\cos\theta$$

for x-z plane, $\phi = 0$

for x-y plane, $\theta = 90°$

$e^{j0} = 1$, $e^{j\pi} = -1$

Prob. 3.13 (cont.)

$$F_a(\theta,\phi) = \left| e^{jkr_1} - e^{jkr_2} - e^{jkr_3} + e^{jkr_4} \right|^2$$

$$= \left| e^{jkr} \left(e^{jk\frac{d}{2}\cos\phi\sin\theta} e^{-jk\frac{d}{2}\cos\theta} - e^{-jk\frac{d}{2}\cos\phi\sin\theta} e^{-jk\frac{d}{2}\cos\theta} \right.\right.$$

$$\left.\left. - e^{jk\frac{d}{2}\cos\phi\sin\theta} e^{jk\frac{d}{2}\cos\theta} + e^{-jk\frac{d}{2}\cos\phi\sin\theta} e^{jk\frac{d}{2}\cos\theta} \right)\right|^2$$

$$= \left| e^{jkr} \right|^2 \cdot \left| e^{-jk\frac{d}{2}\cos\theta} \left(e^{jk\frac{d}{2}\cos\phi\sin\theta} - e^{-jk\frac{d}{2}\cos\phi\sin\theta} \right) - e^{jk\frac{d}{2}\cos\theta} \left(e^{jk\frac{d}{2}\cos\phi\sin\theta} - e^{-jk\frac{d}{2}\cos\phi\sin\theta} \right) \right|^2$$

$$= \left| 2j\sin\left(k\frac{d}{2}\cos\theta\right) \cdot 2\sin\left(k\frac{d}{2}\cos\phi\sin\theta\right) \right|^2$$

$$= 4^2 \sin^2\left(k\frac{d}{2}\cos\theta\right) \cdot \sin^2\left(k\frac{d}{2}\cos\phi\sin\theta\right)$$

Since $d = \frac{\lambda}{2}$, $k\frac{d}{2} = \frac{\pi}{2}$

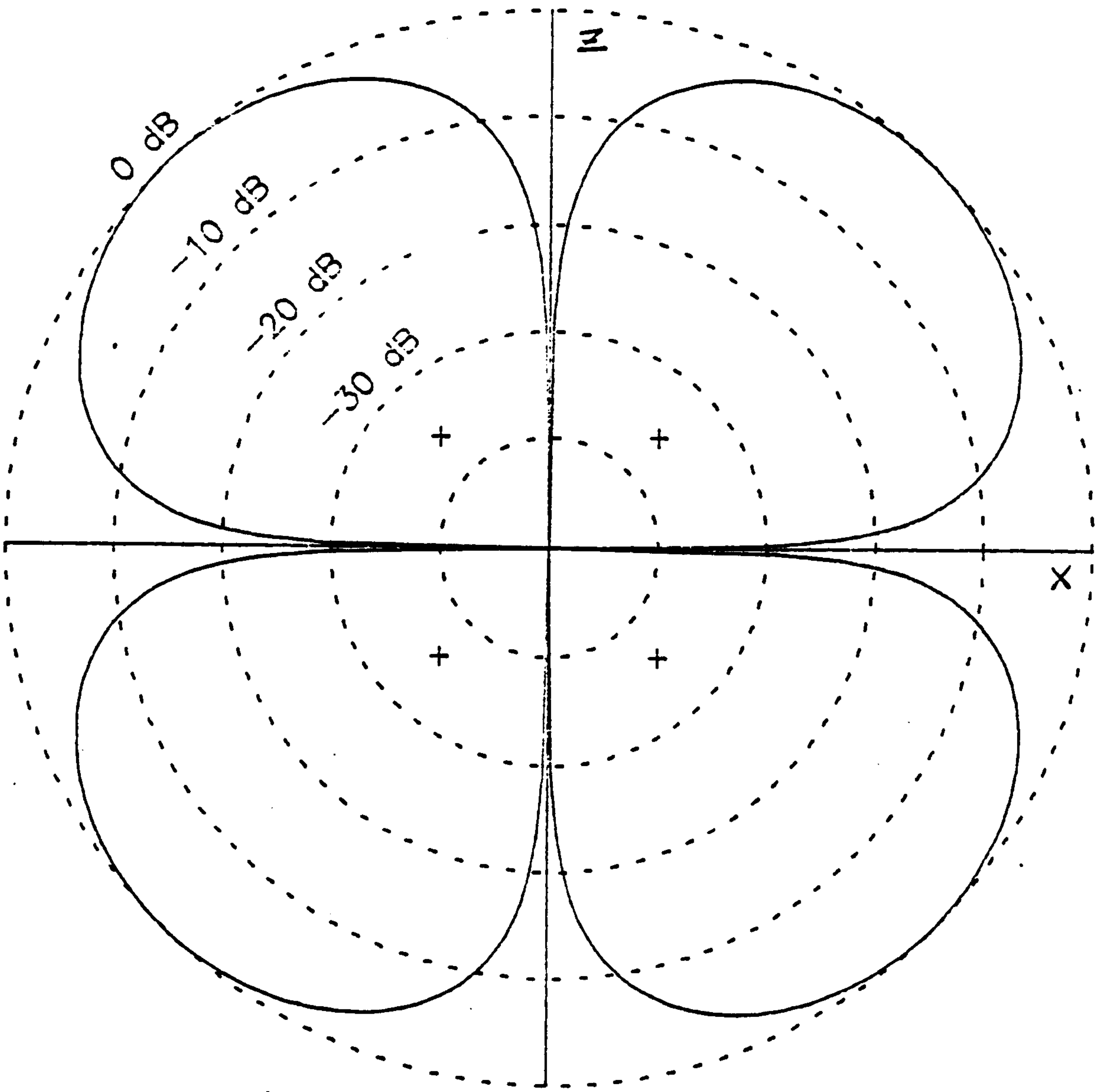
$$F_a(\theta,\phi) = 16\sin^2\left(\frac{\pi}{2}\cos\theta\right) \cdot \sin^2\left(\frac{\pi}{2}\cos\phi\sin\theta\right)$$

$$F_a(\theta,\phi)_{max} = F_a\left(\frac{\pi}{4}, 0\right) = 10.31$$

$$F_{an}(\theta,\phi) = 1.55\sin^2\left(\frac{\pi}{2}\cos\theta\right) \cdot \sin^2\left(\frac{\pi}{2}\cos\phi\sin\theta\right)$$

x-y plane, $\theta = 90^\circ \rightarrow F_{an}(\phi) = 0$

x-z plane, $\phi = 0 \rightarrow F_{an}(\theta) = 1.55\sin^2\left(\frac{\pi}{2}\cos\theta\right) \cdot \sin^2\left(\frac{\pi}{2}\sin\theta\right)$



Plot for Problem 3.13. $F_{an}(\theta)$ versus θ.