CHAPTER 2

Prob. 2.1 Eq. (2.4) $\nabla^2 \bar{E} = \mu\epsilon \dfrac{\partial^2 \bar{E}}{\partial t^2}$

from Eq. (2.5) $\bar{E}(\bar{r},t) = Re\{\bar{E}(\bar{r})\, e^{j\omega t}\}$

time harmonic dependence assumed

$$\nabla^2 \bar{E}(\bar{r},t) = Re\left[\nabla^2 \bar{E}(\bar{r})\, e^{j\omega t}\right]$$

$$\frac{\partial^2 \bar{E}}{\partial t^2} = Re\left[\frac{\partial^2}{\partial t^2}\left(\bar{E}(\bar{r})\, e^{j\omega t}\right)\right] = Re\left[(j\omega)^2 \bar{E}(\bar{r})\, e^{j\omega t}\right]$$
$$= Re\left[-\omega^2 \bar{E}(\bar{r})\, e^{j\omega t}\right]$$

$$\therefore \nabla^2 \bar{E}(\bar{r}) = -\omega^2\mu\epsilon\, \bar{E}(\bar{r})$$

Prob. 2.2 (2.7) $\bar{E}(\bar{r}) = \bar{E}_o \exp\left[\pm j(k_x x + k_y y + k_z z)\right]$

(2.6) $\nabla^2 \bar{E}(\bar{r}) = -\omega^2\mu\epsilon\, \bar{E}(\bar{r})$

$$\nabla^2 \bar{E}(\bar{r}) = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right]\bar{E}(\bar{r})$$

$$= (\pm j)^2 \left(k_x^2 + k_y^2 + k_z^2\right)\bar{E}(\bar{r})$$

from (2.8) $k_x^2 + k_y^2 + k_z^2 = k^2 = \omega^2\mu\epsilon$

$$\therefore \nabla^2 \bar{E}(\bar{r}) = -\omega^2\mu\epsilon\, \bar{E}(\bar{r})$$

2.5 $\bar{E}(\bar{r},t) = \cos(100t + 4x + 2y + 4z)$

$\bar{k} = \hat{x}4 + \hat{y}2 + \hat{z}4$

$|\bar{k}| = [16 + 4 + 16]^{1/2} = 6$

Unit vector in the direction of propagation $\equiv \hat{a}$

where, $\hat{a} = \frac{\bar{k}}{|\bar{k}|} = \hat{x}\frac{2}{3} + \hat{y}\frac{1}{3} + \hat{z}\frac{2}{3}$

2.6 Sea Water $\quad \mu/\mu_0 \cong 1, \quad \epsilon/\epsilon_0 \cong 81$

$\sigma \cong 4.0 \ \mathrm{S\,m^{-1}}$ @ $f = 4 \times 10^8$ Hz

a) Attenuation and phase constants

from (2.16)

$\epsilon_c = \epsilon - j\sigma/\omega \triangleq \epsilon' - j\epsilon''$

from (2.19)

$E_x(z) = E_{x0} \exp(-j\omega\sqrt{\mu\epsilon_c}\, z)$

$$k = \omega\sqrt{\mu\epsilon_c} = \omega\sqrt{\mu_0\epsilon_0}\left(\epsilon_r - j\frac{\sigma}{\omega\epsilon_0}\right)^{1/2}$$

$$\frac{\sigma}{\omega\epsilon_0} = \frac{4}{2\pi \times 4\times10^8 \times 8.85\times10^{-12}} = 180$$

ϵ_r and $\frac{\sigma}{\omega\epsilon_0}$ are of comparable magnitude

must treat sea water as a general lossy medium

2.6 (cont.)

$$k = \frac{\omega}{c}(81 - j180)^{1/2}$$

$$= \frac{2\pi \times 4 \times \cancel{10^8}}{3 \times \cancel{10^8}}\left[197.4 \angle -65.77^\circ\right]^{1/2}$$

$$= 8.378\,(14.05 \angle -32.89^\circ)$$

$$= 98.84 - j63.91$$

$$jk = 63.91 + j98.84$$

Attenuation: $\alpha = 63.91$ nep/m
Phase: $\beta = 98.84$ rad/m

b) for low-loss medium $\omega\epsilon \gg \sigma$
$\omega\epsilon = 1.80$; $\sigma = 4$
$\therefore$ cannot treat as low-loss medium

c) for a conducting medium (high loss) $\sigma \gg \omega\epsilon$
$\therefore$ cannot treat as conducting medium

2.7

	σ (S/m)	ϵ'_r	μ_r
Fresh water	10^{-3}	81	1
wet earth	10^{-3}	15	1
dry earth	10^{-5}	2.5	1

a) $f = 10\,kHz$

Fresh water & WET EARTH $\frac{\sigma}{\omega\epsilon_0} = \frac{10^{-3}}{2\pi \times 10^4 \times 8.85 \times 10^{-12}} = 1798$

dry earth $\frac{\sigma}{\omega\epsilon_0} = \frac{10^{-5}}{2\pi \times 10^4 \times 8.85 \times 10^{-12}} = 17.98$

$\therefore$ Fresh water and wet earth act like conducting media at 10kHz, while dry earth acts like a general lossy medium.

b) $f = 1\,GHz$

Fresh water & wet earth $\frac{\sigma}{\omega\epsilon_0} = \frac{10^{-3}}{2\pi \times 10^9 \times 8.85 \times 10^{-12}} = 0.018$

Dry earth $\frac{\sigma}{\omega\epsilon_0} = 1.8 \times 10^{-4}$

$\therefore$ All three, fresh water, wet earth, and dry earth, act like low-loss media at $f = 1\,GHz$.

2.9 (cont.)

for conducting media, $\alpha = \sqrt{\omega\mu\sigma/2}$

FRESH WATER: $\alpha = [2\pi \times 10^4 \times 4\pi \times 10^{-7} \times 10^{-3} \times \frac{1}{2}]^{1/2}$

$= 2\pi \times 10^{-3}$ nep/m

Attenuation in dB $= 20 \times 2\pi \times 10^{-3} \times \log e$

$= 5.46 \times 10^{-2}$ dB/m

WET EARTH: $\alpha = 2\pi \times 10^{-3}$ nep/m

Attenuation in dB $= 5.46 \times 10^{-2}$ dB/m

for general lossy media, $\alpha = k_o |Im\{\sqrt{\epsilon_r}\}|$

$k_o = \frac{2\pi}{\lambda_o} = \frac{\omega}{c}$

$\omega = 2\pi f = 6.28 \times 10^4$ rad/sec

$k_o = 2.09 \times 10^{-4}$ rad/m

$\epsilon_r = 2.5 - j18 = 18.173 \angle -82.09^\circ$

$\sqrt{\epsilon_r} = 4.263 \angle -41.05^\circ = 3.215 - j2.799$

$Im\{\sqrt{\epsilon_r}\} = -2.799$

$\alpha = 5.85 \times 10^{-4}$ nep/m

Attenuation in dB $= 5.08 \times 10^{-3}$ dB/m

2.10 (cont.)

$$j\omega\sqrt{\mu\epsilon_c} = jk\left[\tfrac{1}{2}\sqrt{1+\tan^2\delta}+1\right]^{1/2}$$
$$+ k\left[\tfrac{1}{2}\sqrt{1+\tan^2\delta}-1\right]^{1/2}$$

$$j\omega\sqrt{\mu\epsilon_c} = \alpha + j\beta$$

$$\therefore \quad \alpha = k\left[\tfrac{1}{2}\sqrt{1+\tan^2\delta}-1\right]^{1/2} \qquad (2.26)$$
$$\beta = k\left[\tfrac{1}{2}\sqrt{1+\tan^2\delta}+1\right]^{1/2} \qquad (2.27$$

2.11

(2.30) $\bar{E}(z,t) = \hat{x}E_{xo}\cos(\omega t-\beta z) + \hat{y}E_{yo}\cos(\omega t-\beta z+\theta)$

may be rewritten in complex form

$$\bar{E}(z,t) = \hat{x}E_{xo}\exp[j(\omega t-\beta z)] + \hat{y}E_{yo}\exp[j(\omega t-\beta z+\theta)] \qquad (1)$$

General form for two circularly polarized waves of opposite sense is

$$\bar{E}(z,t) = \hat{x}A\exp[j(\omega t-\beta z)] + \hat{y}A\exp[j(\omega t-\beta z-90^\circ)]$$
$$+ \hat{x}B\exp[j(\omega t-\beta z)] + \hat{y}B\exp[j(\omega t-\beta z+90^\circ)]$$

$$\bar{E}(z,t) = \hat{x}(A+B)\exp[j(\omega t-\beta z)] \qquad (2)$$
$$+ \hat{y}(-jA+jB)\exp[j(\omega t-\beta z)]$$

2.11 (cont.)

Comparing (1) and (2) gives

$$A + B = E_{xo}$$
$$A - B = jE_{yo}\exp(j\theta)$$

$$\therefore \; A = \tfrac{1}{2}\left[E_{xo} + jE_{yo}\exp(j\theta)\right]$$

$$B = \tfrac{1}{2}\left[E_{xo} - jE_{yo}\exp(j\theta)\right]$$

2.12

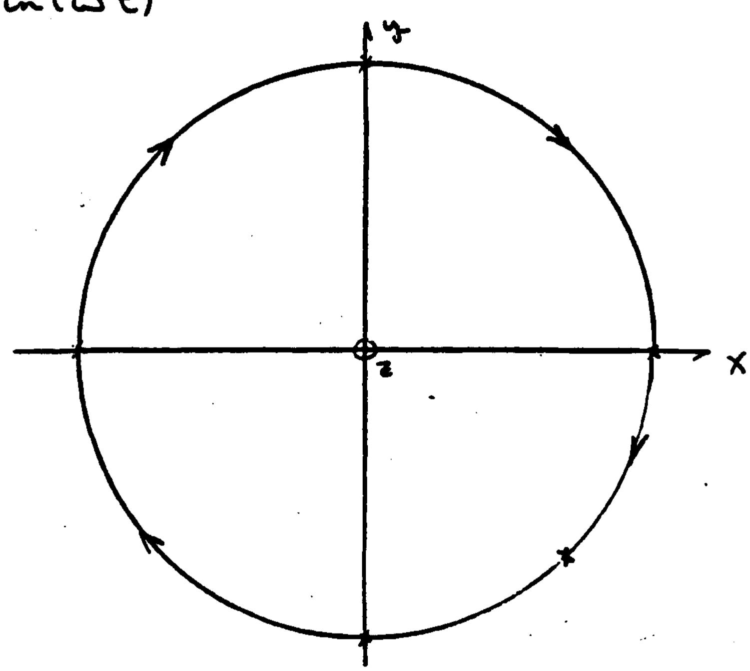
(2.30) $\bar{E}(z,t) = \hat{x}E_{xo}\cos(\omega t - \beta z) + \hat{y}E_{yo}\cos(\omega t - \beta z + \theta)$

$E_{xo} = E_{yo}$, $z = 0$, $\theta = 90^\circ$

$\bar{E}(0,t) = \hat{x}\cos(\omega t) - \hat{y}\sin(\omega t)$

ωt	$\bar{E}\cdot\hat{x}$	$\bar{E}\cdot\hat{y}$
0	1	0
45°	0.707	-0.707
90°	0	-1
180°	-1	0
270°	0	1
360°	1	0

Left hand polarized



Prob. 2.14 (cont)

$$\frac{\cos\Theta_z}{|\eta|} = \frac{\beta/(\alpha^2+\beta^2)^{1/2}}{\omega\mu/(\alpha^2+\beta^2)^{1/2}} = \frac{\beta}{\omega\mu}$$

$$\therefore \bar{S}_a = \frac{\hat{z}}{2} E_0^2 e^{-2\alpha z} \frac{\cos\Theta_z}{|\eta|}$$

Prob. 2.15 $P_t = 10\,W$, $f = 10\,GHz \rightarrow \lambda = 3cm$

$d = 0.57\,m$, $r_m = 1.74 \times 10^3\,km$, $R_m = 3.84 \times 10^5\,km$

antenna beamwidth, $\beta \cong \frac{\lambda}{d}$ (rad)

$\beta = 0.053$ rad $= 3°$

(a) $$|S_r| \cong \frac{P_t}{R^2\beta^2} = \frac{10}{(3.84\times10^5)^2(0.053)^2} = 2.45\times10^{-8}\ W\,km^{-2}$$

(b) AREA OF MOON (CROSS-SECTION), A_m

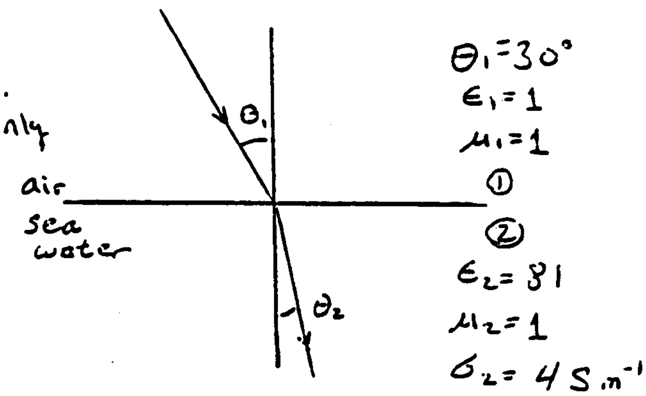
$A_m = \pi r_m^2 = 9.51 \times 10^6\ km^2$

P (intercepted) $= |S_r| \cdot A_m$

$= 0.233\ W$

2.22 horizontal case only

$f = 10\ kHz$



$$\epsilon_r'' = \frac{\sigma}{\omega\epsilon_0}$$

$$\epsilon_r = 81 - j\,7.19 \times 10^6$$

$\epsilon_r'' \gg \epsilon_r' \rightarrow$ conducting medium

$$\alpha = \beta = \sqrt{\omega\mu\sigma/2} \qquad (2.24)$$

$\alpha = 0.397$ nep/m , $\beta = 0.397$ rad/m

a) $\bar{E}_t = \hat{y}\, T_\perp\, E_0 \exp[-j(k_{2x}x - k_{2z}z)]$

$$k_{2x} = k_1 \sin\theta_1$$
$$k_{2z} = k_2 \cos\theta_2$$

$$k_2\cos\theta_2 = \frac{1}{\sqrt{2}}\left\{\left[(p^2+q^2)^{1/2}+q\right]^{1/2} - j\left[(p^2+q^2)^{1/2}-q\right]^{1/2}\right\}$$

where $p = 2\alpha\beta$

$q = \beta^2 - \alpha^2 - k_1^2\sin^2\theta_1$

$p = 0.315$, $q = -(k_1\sin\theta_1)^2$

$$k_1 = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = 2.09 \times 10^{-4}\ \text{rad/m}$$

$k_1 \sin\theta_1 = 1.05 \times 10^{-4}$

$q = -1.10 \times 10^{-8}$

2.22 (cont.)

$$k_2 \cos\theta_2 = 0.397 - j0.397$$

$$T_\perp = \frac{2\eta_2 \cos\theta_1}{\eta_2 \cos\theta_1 + \eta_1 \cos\theta_2} = \frac{2\cos\theta_1}{\cos\theta_1 + \frac{\eta_1}{\eta_2}\cos\theta_2}$$

$$k_2 \cos\theta_2 = \frac{\omega\mu_2}{\eta_2}\cos\theta_2$$

$$k_2 \cos\theta_2 \cdot \frac{\eta_1}{\omega\mu_2} = \frac{\eta_1}{\eta_2}\cos\theta_2$$

$$\frac{\eta_1}{\eta_2}\cos\theta_2 = 0.397(1-j)\frac{377}{2\pi 10^4 \cdot 4\pi \times 10^{-7}}$$
$$= 1896(1-j)$$

$$T_\perp = \frac{1.732}{0.866 + 1896(1-j)} \approx 9.135\times10^{-4}\frac{1}{(1-j)}\cdot\frac{(1+j)}{(1+j)}$$

$$= 4.568\times10^{-4}(1+j)$$

$$\bar{E}_t = \hat{y}\,(4.568\times10^{-4})(1+j)E_0 \exp[0.397z - j(1.05\times10^{-4}x - 0.397z)] \quad \text{for } z<0$$

Real angle of transmission, χ

$$\chi = \tan^{-1}\left[\frac{k_2 x}{\frac{1}{\sqrt{2}}[(p^2+q^2)^{1/2}+q]^{1/2}}\right]$$

$$= \tan^{-1}\left[\frac{1.05\times10^{-4}}{0.397}\right] = 0.0152^\circ$$

2.22 (cont.)

b) $\lambda_2 = ?$

$$k_2 = \frac{2\pi}{\lambda_2} = \sqrt{k_{2x}^2 + |k_{2z}|^2}$$

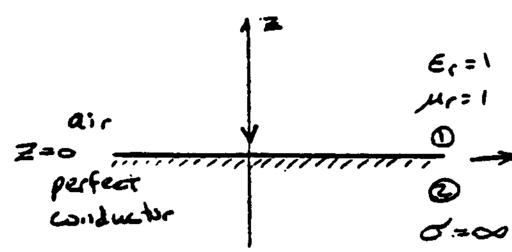
$$k_2 = 0.397$$

$$\lambda_2 = \frac{2\pi}{k_2} = 15.8 \text{ m}$$

c) $v_p = f \cdot \lambda$

$$v_p = (10^4)(15.8) = 158 \text{ km/s}$$

2.23



perfect conductor $\rightarrow \bar{E}_t = 0$

tangential electric field = 0 at z=0

$\bar{E}_i = -\bar{E}_r$ at $z=0$ $\rightarrow R = -1$

from (2.48) and (2.49) with $\theta_1 = 0°$, $R_\perp = R_\parallel = -1$

$T_\perp = T_\parallel = 0$

$$\bar{H}_i = \frac{E_0}{\eta_1} \hat{x} \exp[-jk_1 z]$$

$$\bar{H}_r = \frac{-E_0}{\eta_1}(-\hat{x}) \exp[-jk_1 z]$$

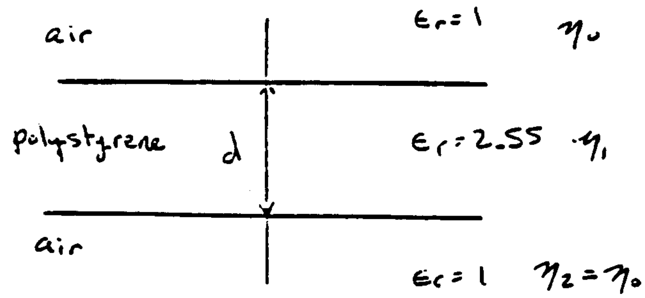
$$\bar{H}_t = 0$$

at $z = 0$, $\bar{H} = \bar{H}_i + \bar{H}_r = \frac{2E_0}{\eta}\hat{x}$, $|\bar{H}(z=0)| = 2|\bar{H}_i|$

2.25 RADOME $f = 5\,\text{GHz}$

$\theta = 0$

want $R = 0$



from 2.24, $R_{01} = \dfrac{\eta_1 - \eta_0}{\eta_1 + \eta_0}$

$$R_{12} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\eta_0 - \eta_1}{\eta_0 + \eta_1} = -R_{01}$$

$$R = \frac{R_{01} + R_{12}\, e^{-2jk_1 d}}{1 + R_{01} R_{12}\, e^{-2jk_1 d}} = \frac{R_{01}\left(1 - e^{-2jk_1 d}\right)}{1 - R_{01}^2\, e^{-2jk_1 d}}$$

for $R = 0 \rightarrow 1 - e^{-2jk_1 d} = 0 \rightarrow e^{-2jk_1 d} = 1$

$2j \frac{2\pi}{\lambda_1} d = j2\pi N \qquad (N = 0, 1, 2, \cdots)$

minimum $d > 0 \rightarrow N = 1$

$d = \lambda_1 / 2$

$\lambda_1 = \lambda_0 / \sqrt{\epsilon_r}$, $\lambda_0 = \dfrac{c}{f} = \dfrac{3 \times 10^8}{5 \times 10^9} = 0.06\ \text{m}$

$\lambda_1 = 6 / \sqrt{2.55} = 3.76\ \text{cm}$

$d = \dfrac{\lambda_1}{2} = 1.88\ \text{cm}$